Example 30 (1.1, p.331: A bargaining breakdown)  There are two people, J and K. J has an asset that he would like to sell to K. J's reservation value is 2 (i.e., he profits only if he sells it for more than 2). Let v denote K's reservation value for the asset. J regards v as drawn from the uniform distribution on [0, 5]. This is the Bayesian Hypothesis, i.e., when a person does not know something, he has probabilistic beliefs about it; the probability distribution models J's uncertainty about the value of v.

Suppose J makes a "take it or leave it" offer to K: J proposes a price p, which K can accept or reject. We can depict this as an extensive form game:

```
  J
    \-----
    \   p
    \-----
    \       
    \    K
    \-----
    \   A
    \-----
    \ \ R
    \--+
   p-2, v-p
    \--+
    \--+
    \   0,0
```

I've assumed here that both players are risk neutral and so their utilities are simply "money plus or minus reservation value," depending on whether the person acquires the item or sells it. I have also assumed that utility is normalized to equal 0 in the case of no trade.

We construct an equilibrium. As argued before, subgame perfection implies that K will accept if

\[ v - p > 0 \iff v > p. \]

The problem of ties (i.e., \( v = p \)) will not matter here because it will occur with probability 0.

J should choose p to maximize his expected profit

\[ (1 - \frac{p}{5}) (p - 2). \]

The \( (1 - p/5) \) is the probability that \( v \in [p, 5] \), in which case trade occurs. The "1/5" reflects the density of the uniform distribution on \([0, 5]\). Taking the derivative with respect to p produces

\[ 0 = -\frac{1}{5} (p - 2) + \left(1 - \frac{p}{5}\right) = -\frac{2p}{5} + \frac{7}{5} \Rightarrow p = \frac{7}{2} = 3.5. \]

The second derivative test verifies that this value of p maximizes J's expected utility.

Besides introducing the Bayesian approach to decision-making, the point of the exam-
ple is the potential inefficiency of the outcome: for values of \( v \in (2, 3.5) \), it is possible for both \( J \) and \( K \) to profitably trade (i.e., gains from trade exist) but these gains are not achieved in the equilibrium that we’ve derived. The inefficiency can be attributed to the incomplete information (i.e., \( J \) does not know what \( K \) will accept). Would it matter if we changed the bargaining game? Yes, but the problem demonstrated with this example has some generality. It forms a major theme in information economics, namely, incomplete information may prevent the attainment of the best possible outcome. This is quite a different theme from the efficiency of trading as taught in Intermediate Microeconomics (302).

How would the problem change if \( J \) and \( K \) were risk averse? \( K \)'s decision will not change, but risk aversion will change \( J \)'s proposal \( \pi \). Suppose his utility function is \( u(w) = \sqrt{w} \). He would then choose \( \pi \) to maximize his expected utility of profit,

\[
(1 - \frac{p}{5}) \sqrt{p - 2}.
\]

Taking the derivative with respect to \( p \) produces

\[
0 = -\frac{1}{5} \sqrt{p - 2} + \frac{1 - \frac{p}{5}}{2\sqrt{p - 2}} \Rightarrow \sqrt{p - 2} = \frac{1 - \frac{p}{5}}{2\sqrt{p - 2}}.
\]

Cross-multiplying produces

\[
2(p - 2) = 5 - p.
\]

\[
3p = 9
\]

\[
p = 3.
\]

0.0.12.1 Alternative Methods of Decision-Making When You Don’t Know Something.

We have adopted the Bayesian Hypothesis, namely, if a person does not know something, then he has probabilistic beliefs about its possible values and makes decisions according to his expected utility. This a big assumption and it does not hold in many practical examples. There are other ways of modeling how individuals make decisions when they are not fully informed.\(^3\) One approach is a worst-case analysis in which one maximizes one’s utility assuming that the unknown turns out to be the worst case from one’s perspective. This is common in engineering. You wouldn’t design a bridge or a levee to withstand the weather of a typical or average day. Instead, you envision the worst possible weather that the structure might be forced to endure during its lifespan and design accordingly.

Let’s be a bit more precise about this. An individual must take an action \( a \in A \). His utility is

\[
u(a, s)
\]

where \( s \in S \) is the state. The individual knows the set \( S \) but does not observe the state \( s \) itself. We think about \( S \) as the range of possibilities. In the bargaining example above, the state \( s \) is \( K \)'s reservation value and \( S = [0, 5] \). In the engineering example, \( S \) would

\(^3\) See Chapter 13 of Luce and Raiffa’s *Games and Decisions* for further discussion.
be the set of all possible weather that the structure might be faced with during its lifespan.

The Bayesian approach assumes that the individual has a probability distribution on $S$ and then chooses $a$ to maximize his expected utility.

The worst-case approach is

$$\max_{a \in A} \min_{s \in S} u(a, s),$$

i.e., different $a$’s are compared under the hypothesis that the worst possible state occurs for each $a$. There is not a single "worst-case" state; a different state may be most challenging for each different $a$. (The weather that most challenges a wooden bridge may not be the same as the weather that most challenges a stone bridge.)

Another approach is **minimizing maximum regret** (also known as **minimizing maximum risk**). The regret associated with a particular action $a'$ in a state $s$ is

$$\max_{a \in A} u(a, s) - u(a', s).$$

In words, it’s the "missed opportunity" from taking the action $a'$ in comparison to the action $a$ that would have been best in that state $s$. The maximum regret associated with a particular action $a'$ is

$$\max_{s \in S} \max_{a \in A} u(a, s) - u(a', s),$$

i.e., it’s the most that the individual might rue or regret from taking the action $a'$.

Minimizing maximum regret selects the action $a'$ to minimize maximum regret, i.e., one evaluates each action’s maximum regret and then chooses the action that has the smallest maximum regret. In a sense, it is a "cover your bases" way of making decisions: you choose your action to minimize your maximum possible loss after the fact.

We’ll mostly deal with the Bayesian approach in this course. Sometimes, however, it is implausible and it is good to know that alternative approaches exist.

**Example 31**  Assume in the above example that $K$ follows his dominant strategy of accepting any $p$ that is less than or equal to his value $v$. What price $p$ does $J$ propose if he takes a maxmin approach? What price $p$ does $J$ propose if he minimizes his maximum regret?

**Maxmin:** When $J$ (the seller) proposes $p$, one of two things happens:

1. $K$ accepts and $J$ gets $p - 2$;
2. K rejects and J gets 0.

The smaller (or the "min") of these two is $p - 2 < 0$ if $p < 2$ and $0$ if $p \geq 2$. We therefore have

$$\max_p \min_v u_J(p, v) = 0,$$

and every $p \geq 2$ is a maxmin strategy (i.e., each $p \geq 2$ guarantees J a payoff of at least 0, which is the most that J can guarantee himself in this game).

As this example illustrates, a weakly dominated strategy (here, $p < 2$) cannot be a maxmin strategy. Maxmin is not particularly helpful in this example in guiding J in the selection of $p$.

Minimizing maximum regret is a bit complicated in this example; we’ll return to it after working some easier examples.

We’ve digressed a bit from our study of auctions to consider alternatives to Bayesian decision-making (i.e., expected utility maximization). Auctions typically involve uncertainty for the seller concerning what the potential buyers are willing to pay for the items for sale. This uncertainty is what motivates the seller to run the auction instead of simply posting a price. A potential buyer may also not know what other potential buyers are willing to pay for the items. The issue is how should we model this incomplete knowledge of the participants. One approach is to make the Bayesian Hypothesis, which is that each participant has probabilistic beliefs about everything that he does not know (e.g., the reservation values of the potential buyers). This is the most common approach to modeling uncertainty in economic theory. It is a big assumption, however, to assume that participants have beliefs that obey the laws of probability. We digress here to consider alternatives to Bayesian decision-making that do not require the assumption of beliefs. These alternative decision criteria are also useful in themselves.

**Example 32** We discussed the maxmin approach last time. Consider the following decision problem:

<table>
<thead>
<tr>
<th>action/state</th>
<th>s1</th>
<th>s2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>$A_2$</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

The maxmin approach selects the action $A_2$. This remains true if the 1 is replaced by 0.000001 and the 100 by 1000000. Some have argued that $A_2$ is reasonable if Nature (the selector of the state) is malicious with the objective of harming the decisionmaker. Is it reasonable if Nature is indifferent?

**Example 33** To analyze this decision problem from the perspective of minimizing maximum regret, we first derive the regret $\max_{a \in A} u(a, s) - u(a', s)$ associated with each action in each state:
The number 99 in the action $A_2$ and state $s_2$ entry, for instance, represents the difference between what the decision maker could have obtained in state 2 (100, by choosing $A_1$) minus what he actually got in state $s_2$ by choosing $A_2$. The maximum regret associated with each of the two possible actions is

<table>
<thead>
<tr>
<th>action</th>
<th>maximum regret</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>1</td>
</tr>
<tr>
<td>$A_2$</td>
<td>99</td>
</tr>
</tbody>
</table>

and so the action $A_1$ minimizes the maximum regret. What is the rationale for picking $A_1$ over $A_2$? If one picks $A_2$ and the state is $s_2$, then it appears after the fact that one made a really bad decision. It is interesting that maxmin and minimizing maximum regret select different actions in this example.

We can apply maxmin and minimax regret in multiplayer games by assuming that a player regards the set of all possible profiles of actions by the other players as the set of states. This can lead to problems in the application of these ideas because the selection of actions by opponents in a game is not the same as a nonintelligence "choice" of Nature of a state; an opposing player in a game presumably makes his choices intelligently and in his own self-interest; solution concepts such as Nash equilibrium may be more sensible in such situations. We apply maxmin and minimax regret to games in which a player does not know enough about the preferences of his opponents in order to deduce how they will behave.

Example 34 An individual is about to retire and has accumulated $2$ million dollars in a tax deferred retirement account such as a 401k. The actions that he might take are his choices of investments for the remainder of his life; a state determines his future date of death and the sequence of investment returns for all possible investment choices between the present and that future date of death. The set of states is very big in this example! How might the individual think about his investment options?

1. If he has significant other wealth, and if he is interested in leaving a large amount of money to his heirs, then he may be interested in maximizing the expected value of his investments at the time of his passing. This is best modeled using the Bayesian approach (the use of the word "expected" is a tip-off).

2. Suppose that the individual is most concerned about these funds being sufficient to support him through the remainder of his life. A maxmin approach may be appropriate, i.e., what investment strategy will guarantee that the investments are sufficient to fund the individual’s support through the remainder of his life? The purchase of an annuity might be an appropriate decision for the individual.

3. The individual measures the performance of his investments each year against the returns of major mutual funds and market indices. He thinks he is a smart guy and can
pick investments for himself, which is contrary to the well-known advice to invest in a portfolio of inexpensively managed mutual funds. He feels really badly if his investment choices do not come close to these statistical measures. This might be modeled as the regret of not having simply bought index funds or the mutual funds themselves. While he may hope to beat these statistical measures, he may also act to insure that he will not be far off from them in any state. This suggests the strategy of minimizing maximum regret.

Example 35  Solve the following decision problem using maxmin and minimax regret.

<table>
<thead>
<tr>
<th>action/state</th>
<th>s1</th>
<th>s2</th>
<th>s3</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>10</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>A2</td>
<td>0</td>
<td>10</td>
<td>4</td>
</tr>
<tr>
<td>A3</td>
<td>5</td>
<td>2</td>
<td>10</td>
</tr>
</tbody>
</table>

For maxmin, we have the worst payoffs for each action as

<table>
<thead>
<tr>
<th>action</th>
<th>worst</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>1</td>
</tr>
<tr>
<td>A2</td>
<td>0</td>
</tr>
<tr>
<td>A3</td>
<td>2</td>
</tr>
</tbody>
</table>

The maxmin criterion selects action A3 because it guarantees the decision maker the largest payoff.

For minimax regret, we have the following regret associated with each action and each state:

<table>
<thead>
<tr>
<th>action/state</th>
<th>regret s1</th>
<th>regret s2</th>
<th>regret s3</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0</td>
<td>5</td>
<td>9</td>
</tr>
<tr>
<td>A2</td>
<td>10</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>A3</td>
<td>5</td>
<td>8</td>
<td>0</td>
</tr>
</tbody>
</table>

Now maximizing over states produces

<table>
<thead>
<tr>
<th>action/state</th>
<th>max. regret s1</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>9</td>
</tr>
<tr>
<td>A2</td>
<td>10</td>
</tr>
<tr>
<td>A3</td>
<td>8</td>
</tr>
</tbody>
</table>

Action A3 minimizes the maximum regret.